

# Spacetime Foam and Dark Energy

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## Abstract.

Due to quantum fluctuations, spacetime is foamy on small scales. The degree of foaminess is found to be consistent with the holographic principle. One way to detect spacetime foam is to look for halos in the images of distant quasars. Applying the holographic foam model to cosmology we "predict" that the cosmic energy density takes on the critical value; and basing only on existing archived data on active galactic nuclei from the Hubble Space Telescope, we also "predict" the existence of dark energy which, we argue, is composed of an enormous number of inert "particles" of extremely long wavelength. We speculate that these "particles" obey infinite statistics.

**Keywords:** spacetime foam, quantum foam, dark energy, holography, infinite statistics, nonlocality  
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## INTRODUCTION AND SUMMARY

Like everything else, spacetime is conceivably subject to quantum fluctuations. So we expect that spacetime, probed at a small enough scale, will appear complicated — something akin in complexity to a turbulent froth that John Wheeler has dubbed "quantum foam," also known as "spacetime foam."<sup>1</sup> But how large are the fluctuations in the fabric of spacetime? To quantify the problem, let us recall that, if spacetime indeed undergoes quantum fluctuations, there will be an intrinsic limitation to the accuracy with which one can measure a distance, for that distance fluctuates. Denoting the fluctuation of a distance  $l$  by  $\delta l$ , on general grounds, we expect  $\delta l \gtrsim l^{1-\alpha} l_P^\alpha$ , where  $l_P = \sqrt{\hbar G/c^3}$  is the Planck length, the characteristic length scale in quantum gravity, and we have denoted the Planck constant, gravitational constant and the speed of light by  $\hbar$ ,  $G$  and  $c$  respectively. The parameter  $\alpha \sim 1$  specifies the different spacetime foam models. In this talk we will concentrate on the model corresponding to  $\alpha = 2/3$ , which has come to be known as the holographic model [2, 3], so called because it is found to be consistent with the holographic principle [4, 5], according to which, the information content inside any three dimensional region of space can be encoded on the two dimensional surface around the region, like a hologram. For comparison, we will also consider the random-walk model [6] corresponding to  $\alpha = 1/2$ .

Contents of this talk: Applying nothing more than quantum mechanics and some rudimentary black hole physics, we derive the holographic model of spacetime foam. Applying the holographic model to cosmology, we "predict" that the cosmic energy

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<sup>1</sup> In the gravitational context, the phenomenon of turbulence is indeed intimately related to the properties of spacetime foam. See Ref. [1].

density takes on the critical value (i.e., the fractional density parameter of the universe  $\Omega \cong 1$ ), consistent with observation. Then aided by some archived data on quasar or AGN from the Hubble Space Telescope, we are led to conclude that dark energy exists. Furthermore we are naturally led to speculate that the constituents of dark energy, unlike ordinary matter, obey an exotic statistics known as infinite statistics in which all representations of the particle permutation group can occur.

## FLUCTUATIONS OF SPACETIME AND MODELS OF SPACETIME FOAM

One way to find out how much a distance  $l$  fluctuates (i.e., the magnitude of  $\delta l$ ) is to carry out a gedanken experiment to measure  $l$ . But, for later use, it is more convenient to find  $\delta l$  by carrying out a process of mapping the geometry of spacetime. This method [7] relies on the fact that quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime. Hence the structure of spacetime foam can be inferred from the accuracy with which we can measure that geometry. Let us consider mapping out the geometry of spacetime for a spherical volume of radius  $l$  over the amount of time  $T = 2l/c$  it takes light to cross the volume. One way to do this is to fill the space with clocks, exchanging signals with other clocks and measuring the signals' times of arrival. This process of mapping the geometry of spacetime is a kind of computation, in which distances are gauged by transmitting and processing information. The total number of operations, including the ticks of the clocks and the measurements of signals, is bounded by the Margolus-Levitin theorem [8] in quantum computation, which stipulates that the rate of operations for any computer cannot exceed the amount of energy  $E$  that is available for computation divided by  $\pi\hbar/2$ . A total mass  $M$  of clocks then yields, via the Margolus-Levitin theorem, the bound on the total number of operations given by  $(2Mc^2/\pi\hbar) \times 2l/c$ . But to prevent black hole formation,  $M$  must be less than  $lc^2/2G$ . Together, these two limits imply that the total number of operations that can occur in a spatial volume of radius  $l$  for a time period  $2l/c$  is no greater than  $2(l/l_P)^2/\pi$ . To maximize spatial resolution, each clock must tick only once during the entire time period. The operations can be regarded as partitioning the spacetime volume into "cells", then on the average each cell occupies a spatial volume no less than  $(4\pi l^3/3)/(2l^2/\pi l_P^2) = 2\pi^2 l l_P^2/3$ , yielding an average separation between neighboring cells no less than  $(2\pi^2/3)^{1/3} l^{1/3} l_P^{2/3}$ . This spatial separation is interpreted as the average minimum uncertainty in the measurement of a distance  $l$ , that is,

$$\delta l \gtrsim l^{1/3} l_P^{2/3}, \quad (1)$$

where and henceforth (with a couple of exceptions) we drop multiplicative factors of order 1. (Recently Gambini and Pullin [9] have derived from first principles, in the framework of loop quantum gravity in spherical symmetry, an uncertainty in the determination of volumes that grows radially, consistent with Eq. (1).)

We can now understand why this quantum foam model has come to be known as the holographic model. Since, on the average, each cell occupies a spatial volume of  $ll_P^2$ , a spatial region of size  $l$  can contain no more than  $l^3/(ll_P^2) = (l/l_P)^2$  cells. Thus this

model corresponds to the case of maximum number of bits of information  $l^2/l_P^2$  in a spatial region of size  $l$ , that is allowed by the holographic principle.

It will prove to be useful to compare the holographic model in the mapping of the geometry of spacetime with the one that corresponds to spreading the spacetime cells uniformly in both space and time. For the latter case, each cell has the size of  $(l^2 l_P^2)^{1/4} = l^{1/2} l_P^{1/2}$  both spatially and temporally so that each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on  $l^{1/2}$  is the hallmark of a random-walk fluctuation, this quantum foam model corresponding to  $\delta l \gtrsim (ll_P)^{1/2}$  is called the random-walk model. Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation. It also yields a smaller bound on the information content in a spatial region, viz.,  $(l/l_p)^2/(l/l_P)^{1/2} = (l^2/l_P^2)^{3/4} = (l/l_P)^{3/2}$  bits.

Note that the minimum  $\delta l$  just found for the holographic model corresponds to the case of maximum energy density  $\rho = (3/8\pi)(ll_P)^{-2}$  (in units  $c = 1 = \hbar$ , and Boltzmann constant  $k_B = 1$  for later use) for a sphere of radius  $l$  not to collapse into a black hole. Hence the holographic model, unlike the other models, requires, for consistency, the energy density to have the critical value. (By contrast, for instance, one can show that the random-walk model corresponds to an energy density  $(ll_P)^{-2} \gtrsim \rho \gtrsim l^{-5/2} l_P^{-3/2}$ .)

## PROBING SPACETIME FOAM AND THE FALL OF THE RANDOM-WALK MODEL

The Planck length  $l_P \sim 10^{-33}$  cm is so short that we need an astronomical (even cosmological) distance  $l$  for its fluctuation  $\delta l$  to be detectable. Let us consider light (with wavelength  $\lambda$ ) from distant quasars or bright active galactic nuclei. Due to quantum fluctuations of spacetime, the wavefront, while planar, is itself "foamy", having random fluctuations in phase  $\Delta\phi \sim 2\pi\delta l/\lambda$  and in the direction of the wave vector given by  $\Delta\phi/2\pi$  (for  $\delta l \ll \lambda$ ). In effect, spacetime foam creates a "seeing disk" whose angular diameter is  $\sim \Delta\phi/2\pi$ . For an interferometer with baseline length  $D$ , this means that dispersion will be seen as a spread in the angular size of a distant point source, causing a reduction in the fringe visibility when  $\Delta\phi/2\pi \sim \lambda/D$ . For a quasar of 1 Gpc away, at infrared wavelength, the holographic model predicts a phase fluctuation  $\Delta\phi \sim 2\pi \times 10^{-9}$  radians. On the other hand, an infrared interferometer (like the Very Large Telescope Interferometer) with  $D \sim 100$  meters has  $\lambda/D \sim 5 \times 10^{-9}$ . Thus, in principle, this method will allow the use of interferometry fringe patterns to test the holographic model! (For more discussion of this proposal to detect spacetime foam, see Ref. [10].)

In the meantime, we can use existing archived data on quasars or active galactic nuclei from the Hubble Space Telescope to test the quantum foam models. Consider, for example, the case of PKS1413+135, an AGN for which the redshift is  $z = 0.2467$ . [11] With  $l \approx 1.2$  Gpc and  $\lambda = 1.6\mu\text{m}$ , we find  $\Delta\phi \sim 10 \times 2\pi$  and  $10^{-9} \times 2\pi$  for the random-walk model and the holographic model of spacetime foam respectively. With  $D = 2.4$  m for HST, we expect to detect halos if  $\Delta\phi \sim 10^{-6} \times 2\pi$ . Thus, the HST image only fails to test the holographic model by 3 orders of magnitude.

However, the absence of a quantum foam induced halo structure in the HST image

of PKS1413+135 rules out convincingly the random-walk model. (In fact, the scaling relation discussed above indicates that all spacetime foam models with  $\alpha \lesssim 0.6$  are ruled out by this HST observation.)

## FROM SPACETIME FOAM TO COSMOLOGY AND "PREDICTION" OF DARK ENERGY

Assuming that there is a unity of physics connecting the Planck scale to the cosmic scale, we can now apply the holographic spacetime foam model to cosmology [7, 12, 13] and henceforth we call that cosmology the holographic foam cosmology (HFC). The fact that our universe is observed to be at or very close to its critical density must be taken as solid albeit indirect evidence in favor of the holographic model because, as discussed above, it is the only model that requires, for its consistency, the maximum energy density without causing gravitational collapse. Specifically, according to the HFC, the cosmic density is

$$\rho = (3/8\pi)(R_H l_P)^{-2} \sim (H/l_P)^2, \quad (2)$$

where  $H$  is the Hubble parameter of the observable universe and  $R_H$  is the Hubble radius, and the cosmic entropy is given by

$$I \sim (R_H/l_P)^2. \quad (3)$$

For the present cosmic era, the energy density is given by  $\rho \sim H_0^2/G \sim (R_H l_P)^{-2}$  (about  $10^{-9}$  joule per cubic meter). Treating the whole universe as a computer, one can apply the Margolus-Levitin theorem to conclude that the universe computes at a rate  $v$  up to  $\rho R_H^3 \sim R_H l_P^{-2}$  ( $\sim 10^{106}$  op/sec), for a total of  $(R_H/l_P)^2$  ( $\sim 10^{122}$ ) operations during its lifetime so far. If all the information of this huge computer is stored in ordinary matter, we can apply standard methods of statistical mechanics to find that the total number  $I$  of bits is  $[(R_H/l_P)^2]^{3/4} = (R_H/l_P)^{3/2}$  ( $\sim 10^{92}$ ). It follows that each bit flips once in the amount of time given by  $I/v \sim (R_H l_P)^{1/2}$  ( $\sim 10^{-14}$  sec). However the average separation of neighboring bits is  $(R_H^3/I)^{1/3} \sim (R_H l_P)^{1/2}$  ( $\sim 10^{-3}$  cm). Hence, assuming only ordinary matter exists to store all the information we are led to conclude that the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above for the case of the random-walk model of spacetime foam.

But, as shown in the previous section, the sharp images of distant quasars or active galactic nuclei observed at the Hubble Space Telescope have ruled out the random-walk model. From the demise of the random-walk model and the fact that ordinary matter only contains an amount of information dense enough to map out spacetime at a level consistent with the random-walk model, one now infers that spacetime is mapped to a finer spatial accuracy than that which is possible with the use of ordinary matter. Therefore there must be another kind of substance with which spacetime can be mapped to the observed accuracy, conceivably as given by the holographic model. The natural conclusion is that unconventional (dark) energy/matter exists! Note that this

argument does not make use of the evidence from recent cosmological (supernovae, cosmic microwave background, and galaxy clusters) observations.

Furthermore, from Eqs. (2) and (3), the average energy carried by each particle/bit of the unconventional energy/matter is  $\rho R_H^3/I \sim R_H^{-1}$  ( $\sim 10^{-31}$  eV). Such long-wavelength (hence “non-local”) bits or “particles” carry negligible kinetic energy. Also according to HFC, it takes each unconventional bit the amount of time  $I/v \sim R_H$  to flip. Thus, on the average, each bit flips once over the course of cosmic history. Compared to the conventional bits carried by ordinary matter, these bits are rather passive and inert (which, by the way, may explain why dark energy is dark).

## DARK ENERGY, HOLOGRAPHY, INFINITE STATISTICS AND NONLOCALITY

What is the overriding difference between conventional matter and unconventional energy/matter (i.e., dark energy and perhaps also dark matter)? To find that out, let us consider a perfect gas of  $N$  particles obeying Boltzmann statistics at temperature  $T$  in a volume  $V$ . For the problem at hand, let us take  $V \sim R_H^3$ ,  $T \sim R_H^{-1}$ , and  $N \sim (R_H/l_P)^2$ . A standard calculation (for the relativistic case) yields the partition function  $Z_N = (N!)^{-1}(V/\lambda^3)^N$ , where  $\lambda = (\pi)^{2/3}/T$ . With the free energy given by  $F = -T \ln Z_N = -NT[\ln(V/N\lambda^3) + 1]$ , we get, for the entropy of the system,  $S = -(\partial F / \partial T)_{V,N} = N[\ln(V/N\lambda^3) + 5/2]$ . The important point to note is that, since  $V \sim \lambda^3$ , the entropy  $S$  becomes nonsensically negative unless  $N \sim 1$  which is equally nonsensical because  $N$  should not be too different from  $(R_H/l_P)^2 \gg 1$ . But the solution is pretty obvious: the  $N$  inside the log in  $S$  somehow must be absent. Then  $S \sim N \sim (R_H/l_P)^2$  without  $N$  being small (of order 1) and  $S$  is non-negative as physically required. That is the case if the “particles” are distinguishable and nonidentical! For in that case, the Gibbs  $1/N!$  factor is absent from the partition function  $Z_N$ , and the entropy becomes

$$S = N[\ln(V/\lambda^3) + 3/2]. \quad (4)$$

Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor is infinite statistics (sometimes called “quantum Boltzmann statistics”). [14, 15]. Thus we have shown that the “particles” constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics. [13]. (Using the Matrix theory approach, Jejjala, Kavic and Minic [16] have also argued that dark energy quanta obey infinite statistics.) What is infinite statistics? Succinctly, a Fock realization of infinite statistics is provided by a  $q$  deformation of the commutation relations of the oscillators:  $a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl}$  with  $q$  between -1 and 1 (the case  $q = \pm 1$  corresponds to bosons or fermions). Two states obtained by acting with the  $N$  oscillators in different orders are orthogonal; i.e., the states may be in any representation of the permutation group.

Infinite statistics appears to have one “defect”: a theory of particles obeying infinite statistics cannot be local. [15]. (That is, the fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.) Remarkably, the TCP theorem and cluster decomposition have been shown to hold despite the lack

of locality. [15]. Actually this lack of locality may have a silver lining. According to the holographic principle, the number of degrees of freedom in a region of space is bounded not by the volume but by the surrounding surface. This suggests that the physical degrees of freedom are not independent but, considered at the Planck scale, they must be infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. Since the holographic principle is believed to be an important ingredient in the formulation of quantum gravity, the lack of locality for theories of infinite statistics may not be a defect; it can actually be a virtue. Quantum gravity and infinite statistics appear to fit together nicely, and the nonlocality present in systems obeying infinite statistics may be related to the nonlocality present in holographic theories.

But there is the question whether cosmic energy density  $\rho \sim H^2/G$  can lead to the accelerating cosmic expansion as observed. Fortunately, it has been pointed out by Zimdahl and Pavon [17] that a transition from an earlier decelerating to a recent and present accelerating cosmic expansion can arise as a pure interaction phenomenon if dark matter is coupled to holographic dark energy with  $\rho \propto H^2$ . As a bonus, within the framework of such cosmological models, we can now understand why, in addition to dark energy, dark matter has to exist. However the phenomenology of holographic foam cosmology has yet to be worked out in detail; further work in this area is warranted.

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